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Fundamental Strategies for Control of a Tethered System in Elliptical Orbits

Noboru Takeichi*
University of Tokyo, Toyko 113-8654, Japan
and
M. C. Natori† and Nobukatsu Okuizumi‡
Institute of Space and Astronautical Science, Kanagawa 229-8510, Japan

Fundamental strategies for libration control and deployment of a tethered system in elliptical orbits are presented. Through the physical interpretations of the dynamic behavior of tethered systems in elliptical orbits, it is shown that the periodic solution is proper as a control objective. An on–off control strategy using a thruster installed to the subsatellite is examined in elliptical orbits, and the periodic on–off control, which acts at certain true anomalies in the orbit, is presented as the fundamental strategy. For tether deployment, uniform rate deployment is examined in elliptical orbits through physical interpretations of the equations of motion and numerical simulations, and it is shown that the slower deployment is preferable to deploy a tether closer to the periodic solution at the end of the deployment. It is considered that the strategies presented in this study can be applied to tethered systems in orbits of an arbitrary eccentricity, as far as the libration is possible.

Nomenclature

e	=	orbital eccentricity
F	=	control thrust force, N
h_{peri}	=	perigee altitude, m
K	=	control gain, s
l	=	length of tether, m
l_{op}	=	operation length of tether, m
m_s	=	mass of subsatellite, kg
n	=	orbital mean motion, rad/s
r	=	orbital radius, m
r_E	=	Earth radius, m
μ	=	Earth gravitational constant, m^3/s^2
ν	=	true anomaly measured from perigee, rad
ψ	=	libration angle, rad

Subscripts

p	=	periodic solution
0	=	initial conditions

Introduction

MANY applications of tethered systems have been proposed because tethers have great potential to realize large-scale space structure systems. A tethered system with multiple subsatellites has been proposed to observe the atmospheric region from 150 to 500 km.[1] The tethered system consists of one large mother satellite and four small observer satellites connected to the mother satellite by one tether, and the system can observe multiple regions simultaneously. Because the purpose of this system is to observe the atmosphere, its orbital speed is reduced by the atmospheric drag; therefore, a high-energy orbit is necessary to execute a long-term operation. Because the observation altitude is limited, the system should take an elliptical orbit. Thus, this tethered system is subjected to various changes of gravity gradient and atmospheric environment.

In the studies on the dynamics of tethered systems in elliptical orbits,[2,3] numerical simulations have shown that the divergence of libration resulting from atmospheric drag can occur, and the tethered system begins tumbling later. It is considered that the tumbling motion of the system can cause operations and scientific observations to be infeasible. It has been concluded that some improvements (e.g., applying thicker tether) or active controls are necessary to lead the tethered system to continue libration for a long-term operation. A deployment strategy to develop the tethered system stably in elliptical orbits must also be considered. The last phase of the deployment naturally gives the initial condition of the operation.

This study aims to investigate and develop the fundamental strategies for deployment and libration control of a tethered system in elliptical orbits. In the past studies only a few strategies have been presented for the libration controls[4,5] and tether deployment[6,7] in elliptical orbits. However, the investigations about the control actuators and their limitations, postdeployment behavior, and the characteristics of the libration in elliptical orbits are insufficient. Therefore, their strategies cannot always be feasible or rational for actual tethered systems. In this study, practical strategies, which can be applied to an arbitrary tethered system, are presented through the sufficient background on the dynamical characteristics. At first, the control objective is discussed through the reconsideration about the previous studies on dynamic behavior of tethered systems in elliptical orbits. For tethered systems in circular orbits, it has been shown that the libration control by thrusters with on–off strategy[8] and the uniform rate deployment[9] are effective and practical for the libration control and the tether deployment, respectively. The effectiveness of these practical strategies in elliptical orbits is investigated through physical interpretations and numerical simulations of a simplified model, which consists of a subsatellite connected by a massless tether to the mother satellite in elliptical orbit. And the fundamental strategies are clearly shown.

Control Objective in Elliptical Orbits

Characteristics of Libration in Elliptical Orbits

In elliptical orbits, tethered systems begin libration and tumbling as a result of the changes of gravity gradient and orbital angular velocity. Periodic solutions of the libration and their stability have been analyzed numerically,[10,11] and it has been shown that the stable libration of a tethered system is possible in elliptical orbits when $e < 0.353$. It is also shown that the tethered system continues the libration stably when the initial condition is in the neighborhood of the

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*Graduate Student, Department of Aeronautics and Astronautics, Graduate School of Engineering; takeichi@taurus.eng.isas.ac.jp.

†Professor, Spacecraft Engineering, 3-1-1 Yoshinodai. Associate Fellow AIAA.

‡Research Associate, Spacecraft Engineering, 3-1-1 Yoshinodai.

periodic solution. The region of the condition, which leads the tethered system to continue the libration, diminishes as the eccentricity increases. Outside of this region, the tethered system begins tumbling motion, which emerges as a chaotic motion.[12] A Poincaré map consists of discrete plots created by sampling the values of states periodically, and it facilitates the understanding of the changes in the motion characteristics of the system over long periods of time. Figure 1 is a Poincaré map showing the values of states at every perigee of numerical integrations with various initial conditions[12]; θ is the attitude angle of the spacecraft measured from the local vertical. In the center of the figure, many closed curves can be seen, and the tethered system continues the libration when the values of states are in this region. There is the periodic solution in the center of the closed curves.

As mentioned before, the libration can be unstable and diverges because of the atmospheric drag, and the total system begins the tumbling motion later.[2,3] The divergence of libration is shown in Fig. 2a. Figure 2b is Poincaré map showing the relative position (m) and the velocity (m/s) between the mother satellite x_m and the subsatellite x in the orbital coordinate, where $-x$ is the flight direction.

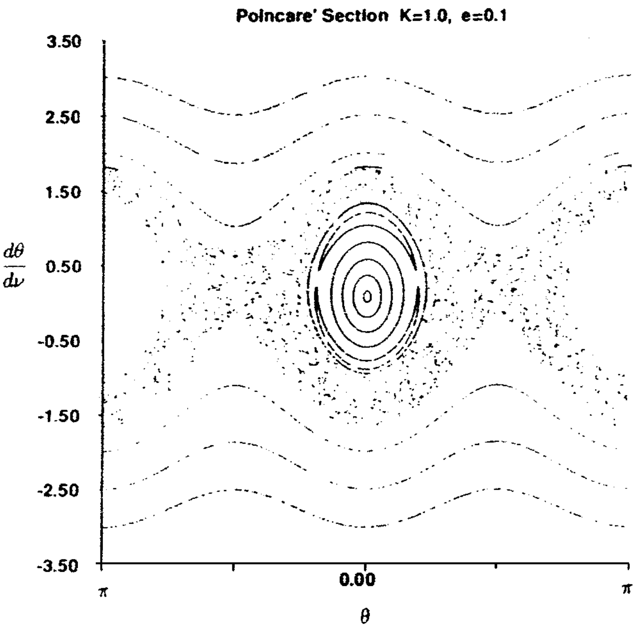
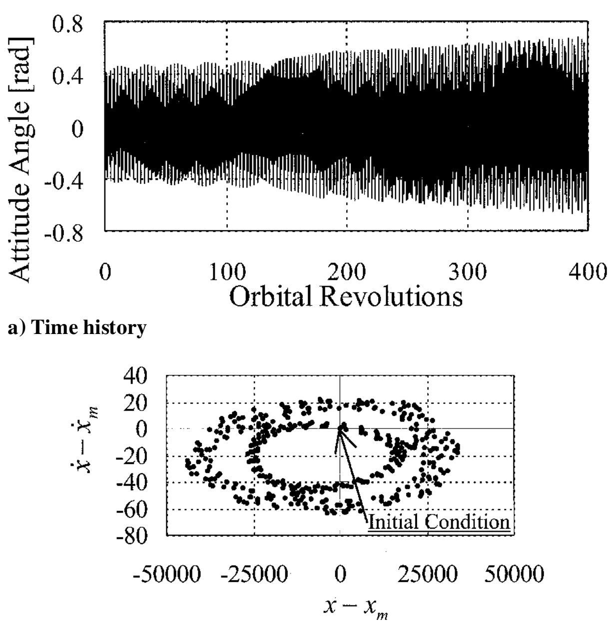


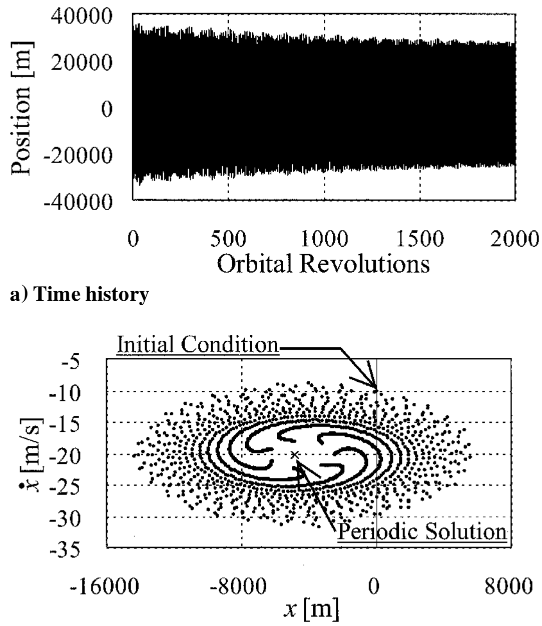
Fig. 1 Poincaré map classifying libration and tumbling motion.[12]



a) Time history

b) Poincaré map

Fig. 2 Divergence of libration.[2]



a) Time history

b) Poincaré map

Fig. 3 Convergence of libration.[13]

In Fig. 2b, the initial condition is given at the origin of both axes. As the libration diverges, the points are plotted outward from the periodic solution. On the other hand, in the study clarifying the cause of divergence,[13] it is shown that the asymptotically stable libration converges on the periodic solution. Figure 3a shows the time history of the position in the x coordinate of the subsatellite, and Fig. 3b is a Poincaré map, where the coordinate is the same as that used in Fig. 2. The physical interpretation of divergence/convergence has been obtained through the comparison with the periodic solutions, and the divergence/convergence of the libration has been interpreted as that of the deviation of the libration from the periodic solution.

Control Objective

For active libration controls it is effective to suppress the divergence of the deviation from the periodic solution and to lead the libration to the periodic solution. In other words, the periodic solution is proper as the control objective of the libration of tethered systems in elliptical orbits.

From the viewpoint of control, the following interpretation can also be valid. The libration of a tethered system in elliptical orbits can be regarded as a sort of periodic system. The control objective of a periodic system should be some periodic solution. In this case, a periodic solution can also mean an equilibrium state. When there is a periodic solution without any controls, it is optimum for the control objective because the necessary control input would be minimum. It has been shown that the tethered system in elliptical orbits has the periodic solution, even when the elasticity of the tether, the out-of-plane motion, and the atmospheric drag is considered.[13] Therefore, the periodic solution is proper as the control objective.

Formulation

System Configurations and Assumptions

This study aims to clarify the fundamental strategies for control of tethered systems in elliptical orbits. For the clarity of the numerical simulations, only the libration and the deployment of the tether are treated, and the effects of the control and the orbital motion are focused. Other factors, such as elasticity, lateral deflection, and damping of the tether, etc., should be ignored, even though they also have effects on the dynamics of the tethered system. The tethered system under consideration is shown in Fig. 4. The following assumptions are made:

1) The attitude motion of the tethered system does not affect the orbital motion.

2) The subsatellite is connected by the tether to the mothersatellite, which coincides to the origin of the orbital coordinate.

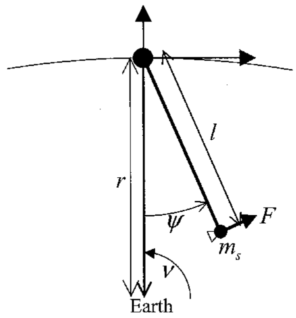


Fig. 4 Coordinate of the tethered system.

3) The center of mass of the system is moving in an equatorial orbit around the spherical Earth.

4) The mass, elasticity, and lateral deflection of the tether are negligible.

5) Only the planar motion is considered.

6) The subsatellite has a thruster, which acts perpendicular to the tether line. The thruster affects only the libration of the tethered system.

Through this model, the libration and the deployment of the tether can be considered, and the effects of the thruster and the orbital motion can be investigated.

Equations of Motion

Equations of motion are as follows including the thruster term modified from those shown in Ref. 6:

$$\ddot{\psi} = -\ddot{\nu} - 2(\dot{l}/l)(\dot{\psi} + \dot{\nu}) - (3\mu/r^3)\cos\psi\sin\psi + F/m_s l \quad (1)$$

The equations of orbital motion are as follows:

$$\ddot{r} = r\dot{\nu}^2 - \mu/r^2 \quad (2)$$

$$\ddot{\nu} = -2\dot{\nu}\dot{r}/r \quad (3)$$

Initial orbital parameters at the apogee are given as follows:

$$r_0 = [(1+e)/(1-e)](r_E + h_{\text{peri}}), \qquad \dot{r}_0 = 0$$

$$\nu_0 = \pi, \qquad \dot{\nu}_0 = \sqrt{(1-e)\mu/r_0^3} \quad (4)$$

System Parameters

In the numerical simulations, the following values are used: $r_E = 6378$ km, $\mu = 3.986 \times 10^{14}$ m^3/s^2, $h_{\text{peri}} = 300$ km, $l_n = 100$ km, $m_s = 500$ kg, and $F = 10$ N. All numerical results are obtained by fourth-order Runge–Kutta method.[14] The numerical accuracy is at least eight digits at the end of numerical integrations.

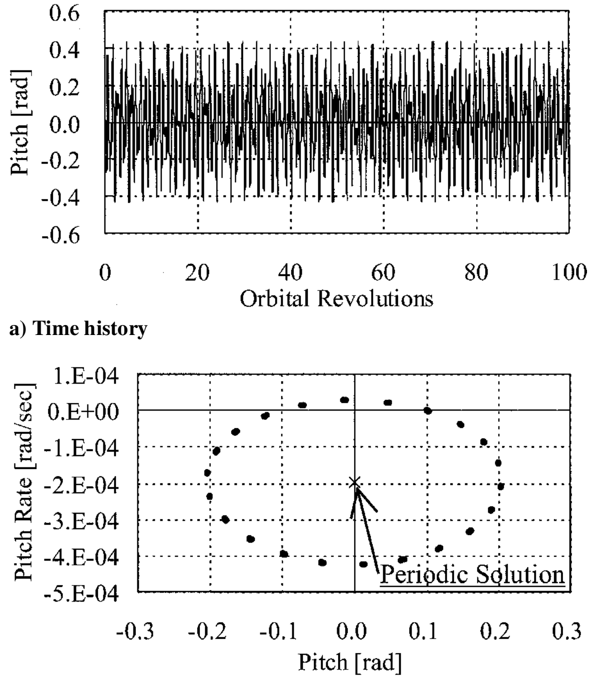
Libration Control

Uncontrolled Motion and Periodic Solution

When neither libration control nor deployment of the tether is considered, Eq. (1) becomes as follows:

$$\ddot{\psi} = -\ddot{\nu} - (3\mu/r^3)\cos\psi\sin\psi \quad (5)$$

Figure 5 shows the result of the numerical integration of Eq. (5) with $\psi_0 = 0.1$ and $\dot{\psi}_0 = 0.0$ in the orbit of $e = 0.2$. In Fig. 5a the amplitude of the libration continues to lie in the fixed limits, and in Fig. 5b all points in the Poincaré map are plotted on a closed curve. This result means that the libration is quasi-periodic and stable—neither converging nor diverging. Figure 6 shows the time history of the pitch angle and the pitch rate of the periodic solution, and this motion is plotted at almost the center of the closed curve of the quasi-periodic libration in the Poincaré map (Fig. 5b). The libration control under the investigation should lead the quasi-periodic libration shown in Fig. 5 to the periodic solution shown in Fig. 6.



a) Time history

b) Poincaré map

Fig. 5 Uncontrolled motion: $e = 0.2$, $(\psi_0, \dot{\psi}_0) = (0.1, 0.0)$.

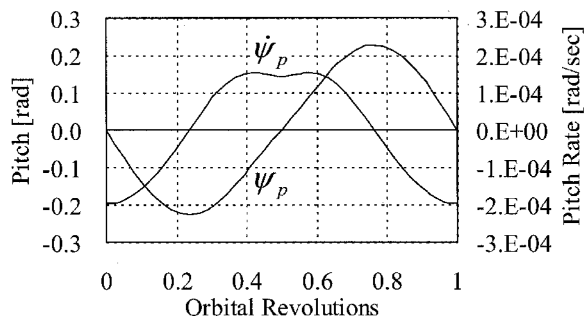


Fig. 6 Periodic solution.

Estimation of Periodic Solution

Because the periodic solution of an actual tethered system depends on its flexibility and orbital environment, such as the atmospheric drag, etc., the periodic solution cannot be exactly obtained in advance. As shown in Figs. 3b and 5b, the periodic solution is plotted at the center of the points of the uncontrolled motion, which is valid at arbitrary true anomaly in the orbit. Therefore, it is considered that the approximate values of states of the periodic solution at a certain true anomaly can be estimated as the mean values of states of the uncontrolled motion for 10–20 orbital revolutions, which are sufficient to make a rough closed curve in the Poincaré map.

Periodic On–Off Control

In the on–off control, the thrust force is assumed to be constant, and the control is made through varying the pulse width and the direction of thrust. An on–off control of some period, which is determined by the time, true anomaly, and some values of states, etc., should be feasible. An on–off control strategy, which has a period determined by the true anomaly, is considered in this study. In this strategy, the pulse width and the direction of thrust are determined according to the difference of the pitch rate from that of the periodic solution. This control adjusts the pitch rate to that of the periodic solution through acceleration/deceleration of the slower/faster libration, respectively. Through this periodic control, the libration becomes closer to the periodic solution as the number of the orbital revolution increases more.

Numerical Simulation

In the numerical simulation, the periodic control at every apogee is considered. It is assumed that the values of states of the periodic solution at the apogee are exactly estimated.

The values of states of the libration are considered to be in the neighborhood of the periodic solution. When the pulse width is short enough to be regarded as a moment compared to the orbital period, the pulse width can be determined linearly proportional to the difference of the pitch rate. The pulse width T_F is determined proportional to the difference between the pitch rate at the apogee and the periodic solution as follows:

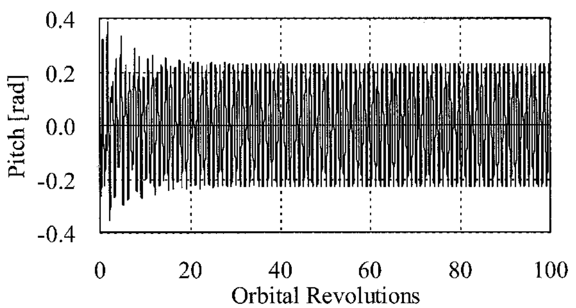
$$T_F = K[(\dot{\psi}_p - \dot{\psi})/n] \tag{6}$$

where n is just to adjust the dimension. When T_F is negative, the thruster acts in the inverse direction for $|T_F|$ seconds. The control strategy just stated is made through the following equations:

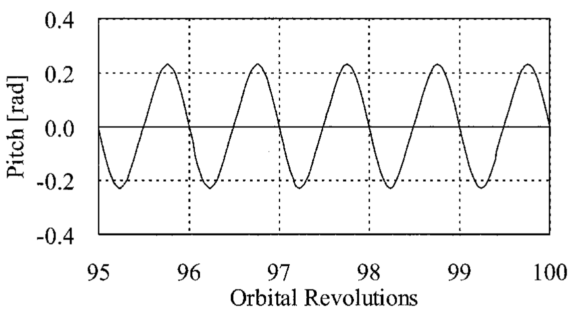
$$\ddot{\psi} = -\ddot{\nu} - (3\mu/r^3)\cos\psi\sin\psi + \mathrm{sgn}(T_F) \times (F/m_s l)$$
$$(t_{\mathrm{apo}} < t < t_{\mathrm{apo}} + |T_F|) \tag{7}$$

$$\ddot{\psi} = -\ddot{\nu} - (3\mu/r^3)\cos\psi\sin\psi$$
$$(t_{\mathrm{apo}} + |T_F| < t < t_{\mathrm{apo}} + T_0) \tag{8}$$

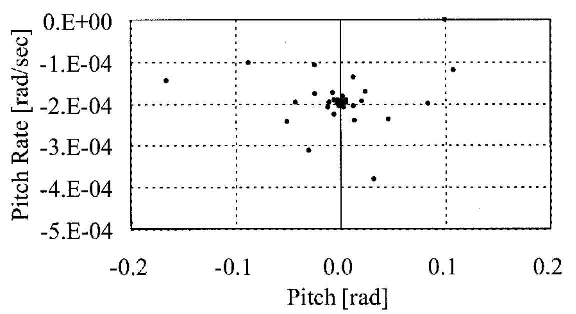
where t_{apo} and T_0 are the passage time at the apogee and orbital period, respectively. Figure 7 shows the numerical result for 100 orbital revolutions in the case of $e = 0.2$ and $K_T = 1000$. The initial condition is $\psi_0 = 0.1$ and $\dot{\psi}_0 = 0.0$ given at the apogee. In the time history (Fig. 7a), the amplitude of the libration converges to be a certain limit, and Fig. 7b shows that the libration coincides with that



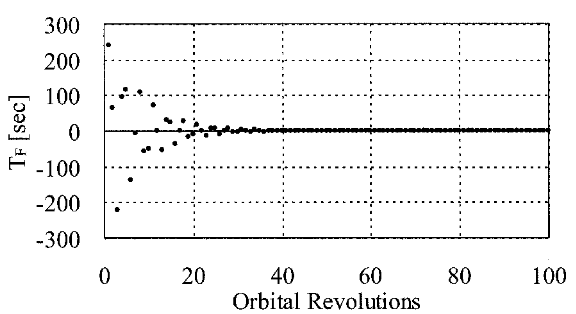
a) Time history (1–100 revs)



b) Time history (95–100 revs)

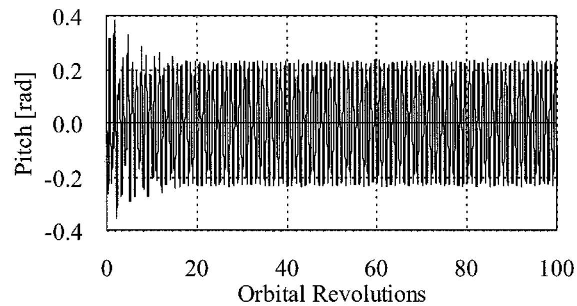


c) Poincaré map

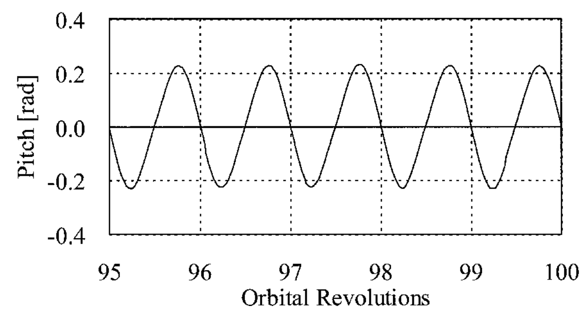


d) History of T_F

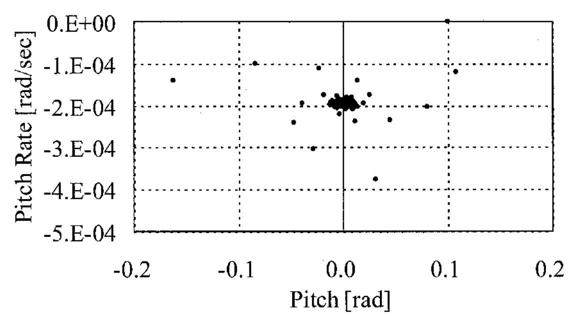
Fig. 7 Controlled motion: $e = 0.2$, $(\psi_0, \dot{\psi}_0) = (0.1, 0.0)$.



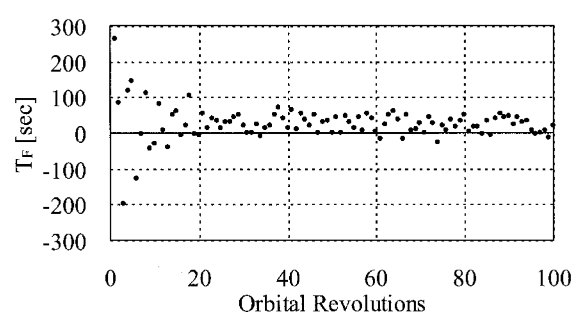
a) Time history (1–100 revs)



b) Time history (95–100 revs)



c) Poincaré map



d) History of T_F

Fig. 8 Controlled motion with measurement error: $e = 0.2$, $(\psi_0, \dot{\psi}_0) = (0.1, 0.0)$.

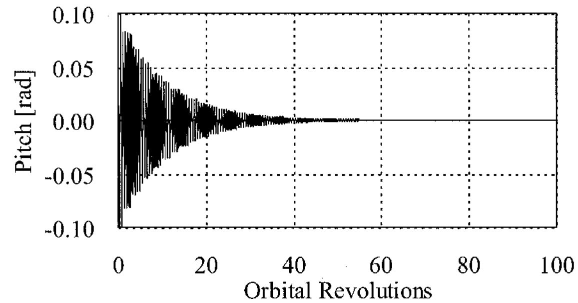
of the periodic solution shown in Fig. 6. The Poincaré map (Fig. 7c) shows the convergence of the plotted points on the periodic solution. It is important that the control input T_F converges on 0 (Fig. 7d), which shows the validity to regard the periodic solution as the control objective.

To investigate the effects of measurement errors, a numerical simulation is carried out. The measurement errors of the values of states and the periodic solution can be included as w in Eq. (9):

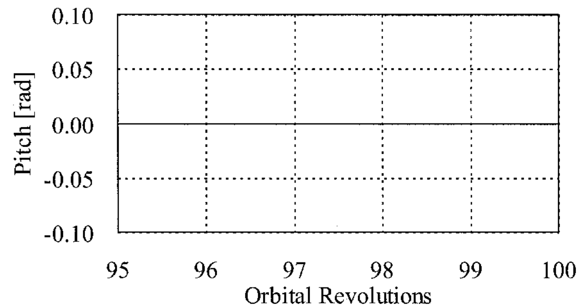
$$T_F = K[(\dot{\psi}_p - \dot{\psi})/n + w] \tag{9}$$

w is given as a nondimensional white noise, which has the average of 0.02 and the variance of 0.02^2, and all other parameters are same as in the preceding section shown in Fig. 7. The result is shown in Fig. 8. It shows that neither the divergence nor the drift of the libration occurs. The libration still continues to stay in the neighborhood of the periodic solution (Fig. 8c), and the effect of the random error on the libration is canceled on average. The bias error affects as a periodic input of a constant value, which does not cause the divergence or the drift of the libration. However, a periodic motion is newly caused as a result of the bias error, which does not coincide with the periodic solution. The control input has a bias and a variance, as shown in Fig. 8d. It is considered that the bias error and the random error cause the bias and the variance of the control input, respectively, and that the measurement errors degrade the efficiency of the control input. It is concluded that the measurement errors do not invalidate the fundamental control and that an accurate measurement must be achieved for better control efficiency.

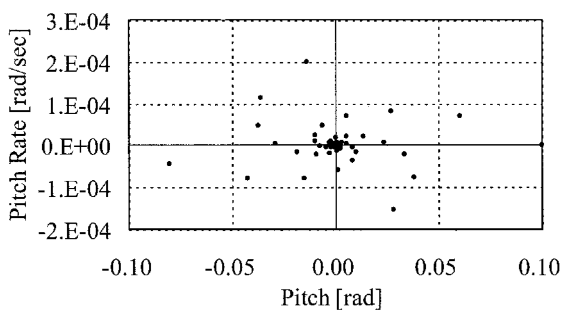
This strategy is also valid for tethered systems in orbits of other eccentricity. The cases of $e = 0.0$, 0.1, and 0.3 are also simulated, and almost the same results are obtained for all of the cases. Figure 9 shows the results of $e = 0.0$. Figure 9a shows that the libration converges on the equilibrium state, which is the periodic solution in circular orbits.



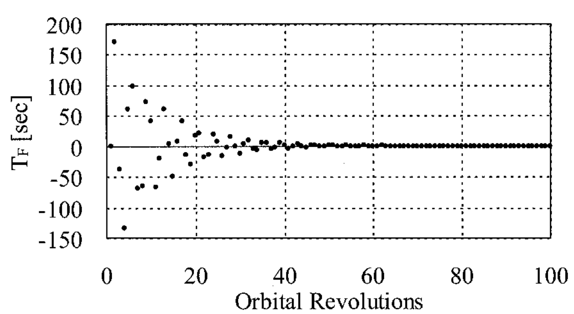
a) Time history (1–100 revs)



b) Time history (95–100 revs)



c) Poincaré map



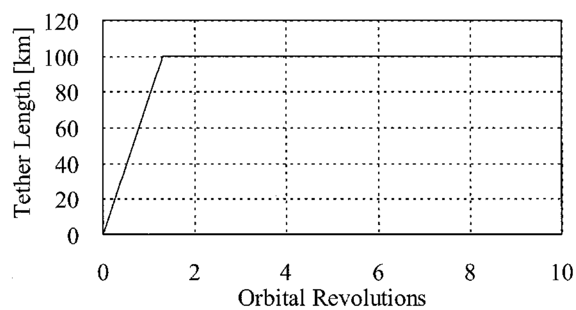
d) History of T_F

Fig. 9 Controlled motion: $e = 0.0$, $(\psi_0, \dot{\psi}_0) = (0.1, 0.0)$.

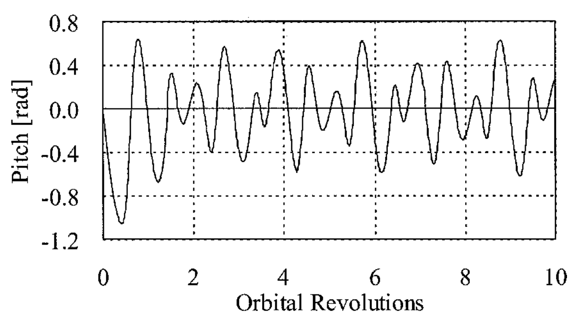
Deployment of Tether

Requirement for Deployment

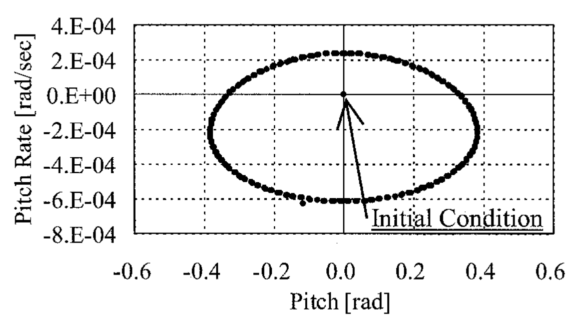
In the preceding section, the strategy for the libration control has been presented, and it is effective to suppress the divergence of the libration when the tethered system has the condition to continue libration at the beginning of the operation. For deployment, it is required that the values of states of the libration have close ones to the periodic solution at the end of the deployment. The dynamical characteristics of the deployment are examined through the physical interpretations of the equations of motion, including the uniform rate deployment and of their numerical integrations.



a) Time history of tether length



b) Time history of libration angle



c) Poincaré map

Fig. 10 Deployment behavior: ($\dot{l}$ = 10 m/s).

Physical Interpretation of Equations of Motion

The equations of motion without the tether deployment can be written from Eq. (5) as follows:

$$\ddot{\psi} + (3\mu/r^3)\cos\psi\sin\psi = -\ddot{\nu} \tag{10}$$

As Fig. 5 shows, the tethered system continues libration stably. This motion can be regarded as a sort of a one-degree-of-freedom forced vibration without any dissipation governed by Eq. (10). The equations of motion with tether deployment are given from Eq. (1) as follows:

$$\ddot{\psi} = -\ddot{\nu} - 2(\dot{l}/l)(\dot{\psi} + \dot{\nu}) - (3\mu/r^3)\cos\psi\sin\psi \tag{11}$$

and this equation can be written as

$$\ddot{\psi} + 2(\dot{l}/l)\dot{\psi} + (3\mu/r^3)\cos\psi\sin\psi = -\ddot{\nu} - 2(\dot{l}/l)\dot{\nu} \tag{12}$$

The tether deployment term $2(\dot{\psi} + \dot{\nu})\dot{l}/l$ can be regarded as a damping term $2\dot{l}\dot{\psi}/l$ and an excitation term $-2\dot{l}\dot{\nu}/l$. When the deployment rate is uniform and $l_0 = 0$ m the following equation is valid:

$$l = \dot{l}t \rightarrow \dot{l}/l = 1/t \tag{13}$$

Then, Eq. (12) can be written as follows:

$$\ddot{\psi} + 2(1/t)\dot{\psi} + (3\mu/r^3)\cos\psi\sin\psi = -\ddot{\nu} - 2(1/t)\dot{\nu} \tag{14}$$

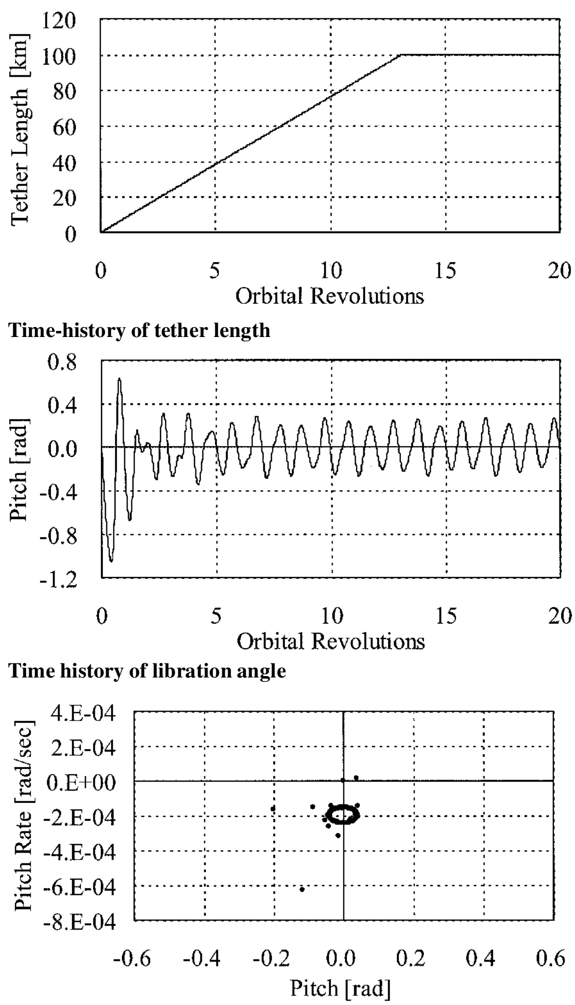
This equation means that the libration does not depend on the deployment rate until the end of the deployment. After introducing Φ, which is a periodic function of one orbital period and has the mean value of zero, the following equation is obtained:

$$\ddot{\psi} + 2(1/t)\dot{\psi} + (3\mu/r^3)\cos\psi\sin\psi = \Phi_1 - 2(1/t)\Phi_2 - 2(1/t)n \tag{15}$$

$2\dot{\psi}/t$ is a damping term, $\Phi - 2\Phi_2/t$ is a periodic excitation term, and $2n/t$ is a term concerning the shift of the center of vibration. Because the deployment time has the limitation as $t < l_{op}/\dot{l}$, the shift of the center of vibration at the end of the deployment does not exactly coincide to the periodic solution. When the deployment is carried out in sufficient duration of time, the system is well damped, and the shift of the center of vibration becomes sufficiently small. Therefore, it can be considered that the values of states of the libration at the end of deployment should be sufficiently close to the periodic solution. It can be concluded that the slower deployment is more preferable as far as the deployment rate is practical.

Numerical Simulation

To examine the effects of the deployment rate on the values of states at the end of deployment, numerical simulations of Eq. (11) have been carried out considering the three deployment rates (10, 1.0, and 0.1 m/s) to deploy a 100-km tether. The orbital eccentricity is set at $e = 0.2$. The initial condition is given as $\psi_0 = 0.0$ and $\dot{\psi}_0 = 0.0$ at the apogee, and the numerical results have been obtained for 200 orbital revolutions, which is sufficient to investigate the postdeployment behavior. Figures 10–12 show the time histories of the tether length and the libration angle and the Poincaré maps for different three deployment rates. Each Poincaré map shows the values of states at the apogee during the 200 orbital revolutions.

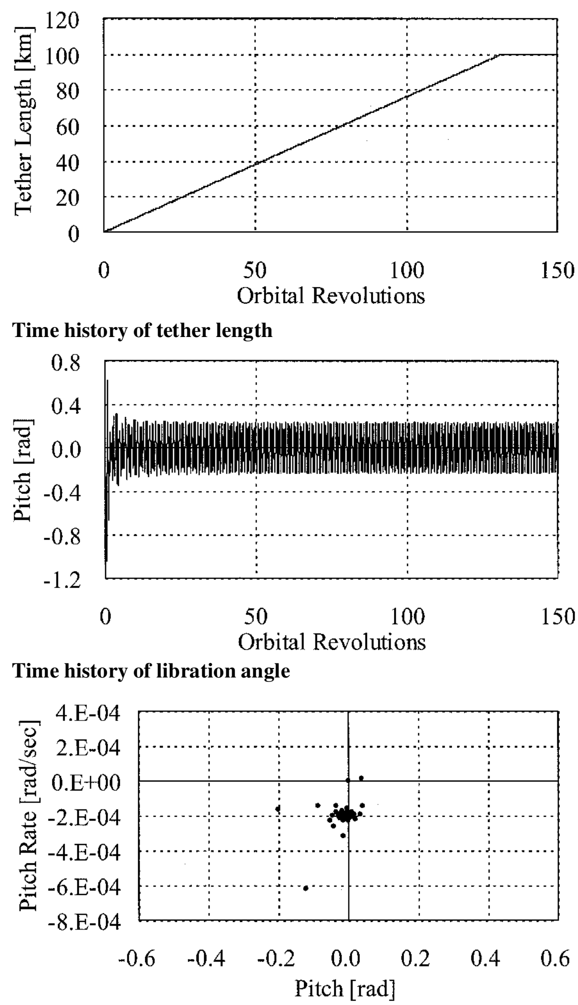


Time-history of tether length

Time history of libration angle

Poincaré map

Fig. 11 Deployment behavior: ($\dot{l}$ = 1 m/s).



Time history of tether length

Time history of libration angle

Poincaré map

Fig. 12 Deployment behavior: ($\dot{l}$ = 0.1 m/s).

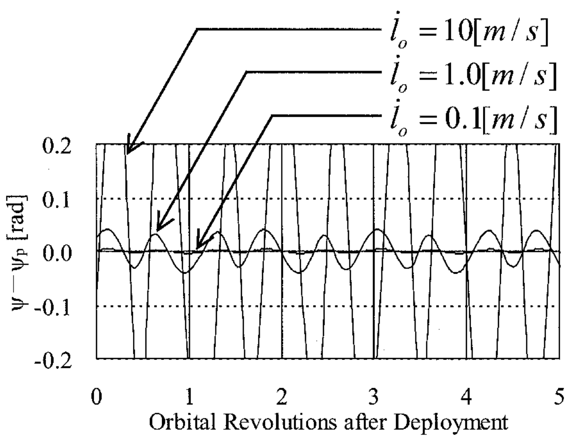


Fig. 13 Difference from periodic solution after deployment.

Figure 13 shows the difference of the libration angle from that of the periodic solution for five revolutions just after the end of deployment. From the Poincaré maps and Fig. 13, it is obvious that the values of states of the libration are closer to the periodic solution, when the deployment is slower. In the case of $\dot{l} = 10$ m/s, the deployment time is so short that the damping hardly affects the libration compared with the other two cases. Therefore, the values of states of the libration after the deployment are further from the periodic solution than the initial condition as shown in Fig. 10c. On the other hand, in the cases of $\dot{l} = 1.0$ and 0.1 m/s, it can be considered that the effect of the damping is large enough for the libration to follow the shift of the center of vibration.

These numerical results agree with the interpretations of Eq. (15). From the practical viewpoint, it is considered that the uniform rate deployment of $\dot{l} = 1.0$ m/s can lead the libration of a 100-km tether close enough to the periodic solution at the end of the deployment.

Conclusions

Fundamental strategies for the libration control and the deployment of tethered systems in elliptical orbits are presented.

It is shown that the periodic solution of the libration is proper as the control objective and that the strategies for tethered systems in circular orbits are extended to the system in elliptical orbits.

The following conclusions are obtained in detail:

1) Through the physical interpretations on dynamics of tethered systems in elliptical orbits, it is shown that the periodic solution, which has the same period as one orbital period, is proper as the control objective for libration controls.

2) The on–off control by the thruster installed to the subsatellite is examined as a fundamental strategy, and the periodic on–off control at a certain true anomaly in the orbit is presented as a simple and practical strategy. Its validity is shown through the numerical simulations of a simplified model.

3) Through physical interpretations of the equations of motion including the tether deployment, the characteristics of the uniform rate deployment are examined. When the deployment time is sufficiently long, the effect of the damping on the libration becomes large, and the shift of the center of vibration becomes small. Therefore, the deployments of small rate are preferable to deploy a tether stably.

In the numerical simulations, elasticity, lateral deflection, damping, and out-of-plane motions are ignored for the clarity of the study. However, these factors also have effects on the dynamics, and some can be harmful for the fundamental controls. For example, the libration control can pump the higher-order transverse mode of the tether, and the out-of-plane libration of large amplitude can invalidate the control strategies. Therefore, the effects of these factors must be clarified in future works. Some optimization of the measurement and the control law should also be investigated in future works.

It is considered that the strategies presented in this study can be applied to tethered systems in orbits of an arbitrary eccentricity, as far as the libration is possible. Many other strategies for the libration control, the tether deployment, and the tether retrieval can also be applied to tethered systems in elliptical orbits, which should be treated in future works.

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D. B. Spencer
Associate Editor